Defining $H'(x) = \log Z(H) + H(x)/T$, also a Walsh expression of degree d, and applying Theorem 1 with H' and $g(x) = \exp(-x)$ yields that all information on π is contained in the correlations of degree $\leq d$. This is equivalent to the statement that a dth-order Boltzmann machine without hidden units is uncapable of capturing correlations of degree $> d$, a well-known "folk-theorem," unproven until now.

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The Commutativity of Up/Downsampling in Two Dimensions

Jelena Kovačević and Martin Vetterli

Abstract —**It is shown under which conditions up- and downsampling can be interchanged in two dimensions. This is the generalization to arbitrary two-dimensional lattices of the result that one-dimensional up- and downsampling commute *iff* their sampling rates are coprime.**

Index Terms —**Multirate processing, multidimensional sampling.**

I. Introduction

It is known that in one dimension we can interchange up- and downsampling *if and only if* their sampling rates are coprime [1]. To the authors' knowledge in two dimensions the problem has been open until now. When the two-dimensional sampling is separable, the extension of the result is trivial. The interesting case appears when the two-dimensional sampling is represented by arbitrary lattices. Thus conditions under which the commutativity can be achieved are more complex and are closely related to the notion of the *greatest common sublattice* of the sampling lattices in question. In this correspondence, after some preliminaries, we state and prove a theorem solving the problem of commutativity in two dimensions. Some illustrative examples are given in Figs. 3(a)–3(c).

One of the possible applications of the result would be in building multirate filter banks with rational sampling rate changes [2]. In [2] a direct method for designing filter banks with arbitrary rational sampling rate changes was given, which as its key element uses the result on commutativity. There it was shown that, in order to avoid designing one filter bank which

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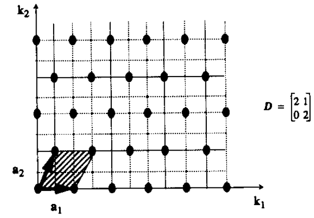

Fig. 1. Hexagonal lattice Λ together with its unit cell.

would divide the spectrum into a number of parts and the other one which would resynthesize the appropriate subspectrums so as to get fractional parts, one had to interchange upsampler with a downsampler in each branch. Note that throughout the paper $\text{lcm}(a, b)$ will denote the *greatest common multiple* of a and b and $\gcd(a, b)$ their *greatest common divisor*; bold letters will denote vectors and matrices.

II. Some Results from the Theory of Lattices

This section presents some basic concepts from lattice theory [3].

Definition 1: Let $\boldsymbol{a}_1, \boldsymbol{a}_2$ be two linearly independent real vectors in two-dimensional real Euclidean space $\mathscr{R}^2$. A *lattice* Λ in $\mathscr{R}^2$ is the set of all linear combinations of $\boldsymbol{a}_1, \boldsymbol{a}_2$ with integer coefficients:

$$\Lambda = \{\lambda_1 \boldsymbol{a}_1 + \lambda_2 \boldsymbol{a}_2, \lambda_1, \lambda_2 \in \mathscr{Z}\}. \tag{1}$$

If $\boldsymbol{D}$ is a matrix with columns $\boldsymbol{a}_1, \boldsymbol{a}_2$, then a lattice is the set of all vectors generated by $\boldsymbol{D} \cdot \boldsymbol{n}, \boldsymbol{n} \in \mathscr{Z}^2$. Since the elements of $\boldsymbol{D}$ belong to $\mathscr{Z}$, which is a principal ideal ring, *unimodular matrices* would be all those with determinant equal to ± 1 [4]. In what follows all matrices involved will be integer matrices. Note that a basis for a lattice is not uniquely determined since $\boldsymbol{D} \cdot \boldsymbol{V}$ with unimodular $\boldsymbol{V}$ is again a basis for Λ, while $d(\Lambda) = |\det(\boldsymbol{D})|$ is unique, and physically represents the reciprocal of the sampling density [5]. Thus, for example, $\mathscr{Z}^2$ is the lattice generated by a 2×2 identity matrix $\boldsymbol{I}$ that corresponds to the standard orthonormal basis. Since only $d(\mathscr{Z}^2)$ is unique, it follows that $\mathscr{Z}^2$ can be generated by any unimodular $\boldsymbol{D}$.

Definition 2: If every point of lattice Λ is also a point of lattice M, then we say that Λ is a *sublattice* of M.

The determinant of Λ is then an integer multiple of the determinant of M.

Definition 3: Let Λ_1 and Λ_2 be lattices. The *greatest common sublattice* of Λ_1 and Λ_2, denoted by $\text{gcs}(\Lambda_1, \Lambda_2)$, is the set of all points belonging to both Λ_1 and Λ_2 [6], i.e.,

$$\text{gcs}(\Lambda_1, \Lambda_2) = \Lambda_1 \cap \Lambda_2. \tag{2}$$

Then it is obvious that $d(\text{gcs}(\Lambda_1, \Lambda_2)) = k_1 d(\Lambda_1) = k_2 d(\Lambda_2)$. It is often useful to choose a basis so as to have a simple form of $\boldsymbol{D}$. It can be shown [3] that $\boldsymbol{D}$ can always be uniquely represented as

$$\boldsymbol{D} = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}, \tag{3}$$

where $a > 0$, $d > 0$, and $0 \leq b < a$. This representation will be used throughout this correspondence. A *unit cell* U will denote a set of points belonging to the parallelogram formed by the two basis vectors $\boldsymbol{a}_1, \boldsymbol{a}_2$. Note that it contains exactly $\det(\boldsymbol{D})$ points.

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$p_{n-1} + p_n = 2^{-j}$, where j is a nonnegative integer. To prove this, we only have to show that under this assumption, there exists a source distribution such that $\underline{R}_1 = R_{\text{Huff}}$. It is easy to see that

$$= H_b\left(p_{n-k+1}^{(k-1)}\right) - H_b\left(p_{n-k}^{(k)}\right) + \left[p_{n-k}^{(k-1)} + p_{n-k+1}^{(k-1)}\right]$$

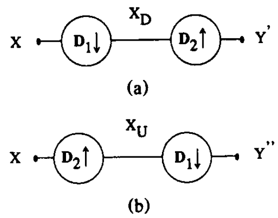

Fig. 2. (a) Downsampler followed by upsampler. (b) Upsampler followed by downsampler.

Fig. 1 shows an example of a lattice together with its unit cell. Also, in what follows a *rectangular lattice* is the one with $b = 0$ in (3) while a *quadratic* one is rectangular with $a = d$. Now, we state a proposition showing how to change a basis for $\mathcal{G}^2$ so as to transform any lattice into a rectangular one.

Proposition 1: Any lattice generated by $\boldsymbol{D}$ can be represented as a rectangular lattice in $\mathcal{G}^2$ generated by some unimodular $\boldsymbol{I}_1$.

Proof: A matrix whose elements belong to a principal ideal domain possesses a Smith normal form [4], i.e., it can be written as

$$\boldsymbol{D} = \boldsymbol{I}_1 \boldsymbol{\Lambda} \boldsymbol{I}_2, \tag{4}$$

where $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ are unimodular and $\boldsymbol{\Lambda}$ is a diagonal matrix. Rewrite (4) as

$$\boldsymbol{D}\boldsymbol{I}_2^{-1} = \boldsymbol{I}_1 \boldsymbol{\Lambda}. \tag{5}$$

Since $\boldsymbol{I}_2$ is unimodular, $\boldsymbol{I}_2^{-1}$ is unimodular as well, and thus $\boldsymbol{D}\boldsymbol{I}_2^{-1}$ still represents the same lattice. Now (5) is exactly what we want since the desired rectangular latice is represented by $\boldsymbol{\Lambda}$ and $\boldsymbol{I}_1$ is the matrix containing the nonstandard basis for $\mathcal{G}^2$. □

III. The Commutativity of Up/Downsampling

Let us briefly recall the one-dimensional result [1]: Upsampling by a factor of N_2 and downsampling by a factor of N_1 can be interchanged *if and only if* N_1 and N_2 are relatively prime. Going back to the original problem, let us consider Figs. 2(a) and 2(b). $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are matrices corresponding to the downsampling and upsampling lattices $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$ respectively. Note that U_1 will denote the unit cell of the first lattice. In the case where downsampling by $\boldsymbol{D}_1$ comes first, the Fourier transform of the signal at the output can be expressed as [5]:

$$Y'(\boldsymbol{\Omega}) = X_D(\boldsymbol{D}_2^t \boldsymbol{\Omega}) = \frac{1}{N_1} \sum_{\boldsymbol{k} \in U_1} X\left[(\boldsymbol{D}_1^t)^{-1} \boldsymbol{D}_2^t \boldsymbol{\Omega} - 2\pi (\boldsymbol{D}_1^t)^{-1} \boldsymbol{k}\right], \tag{6}$$

where X_D is the signal after downsampling, $\boldsymbol{\Omega}$ is a two-dimensional frequency vector, and $N_1 = \det(\boldsymbol{D}_1)$. If $\boldsymbol{D}_2$ came first instead, we would have

$$\begin{aligned} Y''(\boldsymbol{\Omega}) &= \frac{1}{N_1} \sum_{\boldsymbol{k} \in U_1} X_U\left[(\boldsymbol{D}_1^t)^{-1}(\boldsymbol{\Omega} - 2\pi \boldsymbol{k})\right] \\ &= \frac{1}{N_1} \sum_{\boldsymbol{k} \in U_1} X\left[\boldsymbol{D}_2^t (\boldsymbol{D}_1^t)^{-1} \boldsymbol{\Omega} - 2\pi \boldsymbol{D}_2^t (\boldsymbol{D}_1^t)^{-1} \boldsymbol{k}\right], \end{aligned} \tag{7}$$

where X_U is the signal after upsampling. In order to have $Y'(\boldsymbol{\Omega}) = Y''(\boldsymbol{\Omega})$, we must first ensure that the resulting matrices next to $\boldsymbol{\Omega}$ are the same, i.e., $\boldsymbol{D}_2$ and $\boldsymbol{D}_1^{-1}$ have to commute. This leads to the following combinations of possible up/downsampling matrices (assuming $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are of the form (3) with corresponding subscripts):

1) $b_1 = 0 \wedge a_1 = d_1$, i.e., $\boldsymbol{D}_1$ is quadratic and $\boldsymbol{D}_2$ is arbitrary,
2) $b_1 = 0 \wedge b_2 = 0 \wedge a_1 \neq d_1$, i.e., $\boldsymbol{D}_1$ is rectangular and $\boldsymbol{D}_2$ is rectangular or quadratic,
3) $b_1 \neq 0 \wedge d_2 = a_2 + b_2(d_1 - a_1)/b_1$, i.e., $\boldsymbol{D}_1$ is arbitrary nonrectangular and $\boldsymbol{D}_2$ is arbitrary with the given constraint on d_2.

Next we have to find when the set of vectors generated in (6) with $\boldsymbol{k} \in U_1$ is equivalent to the set of vectors generated in (7) with again $\boldsymbol{k} \in U_1$. Calling these two sets A and B, we have

$$A = \left\{e^{-2\pi j (\boldsymbol{D}_1^t)^{-1} \boldsymbol{k}},\ \boldsymbol{k} \in U_1\right\}, \tag{8}$$

$$B = \left\{e^{-2\pi j \boldsymbol{D}_2^t (\boldsymbol{D}_1^t)^{-1} \boldsymbol{n}},\ \boldsymbol{n} \in U_1\right\}. \tag{9}$$

Equivalence of these sets is exactly the possibility of interchanging an upsampler with a downsampler. Note that A contains exactly N_1 distinct elements, i.e.,

$$\forall \boldsymbol{k}, \boldsymbol{n} \in U_1, \qquad \boldsymbol{k} \neq \boldsymbol{n} \Rightarrow e^{-2\pi j (\boldsymbol{D}_1^t)^{-1} \boldsymbol{k}} \neq e^{-2\pi j (\boldsymbol{D}_1^t)^{-1} \boldsymbol{n}}.$$

Now we are ready to state the following theorem.

Theorem 1: Assuming $(\boldsymbol{D}_1^t)^{-1}$ and $\boldsymbol{D}_2^t$ commute, an upsampler and a downsampler are interchangeable (i.e., the sets A and B just defined are equivalent) iff the determinant of the greatest common sublattice of $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$ equals the product of the determinants of $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\Lambda}_2$, i.e.,

$$\forall \boldsymbol{k}, \boldsymbol{n} \in U_1, \qquad \boldsymbol{k} \neq \boldsymbol{n} \Rightarrow e^{-2\pi j \boldsymbol{D}_2^t (\boldsymbol{D}_1^t)^{-1} \boldsymbol{k}} \neq e^{-2\pi j \boldsymbol{D}_2^t (\boldsymbol{D}_1^t)^{-1} \boldsymbol{n}} \quad (*)$$

$$\Leftrightarrow$$

$$d(\mathrm{gcs}(\Lambda_1, \Lambda_2)) = d(\Lambda_1)\, d(\Lambda_2). \qquad (**)$$

The proof of the Theorem is given in the Appendix A.

As an illustration, three examples are given in Figs. 3(a)–3(c). The first one shows what happens if the matrices $(\boldsymbol{D}_1^t)^{-1}$ and $\boldsymbol{D}_2^t$ do not commute. The second one is an instance where the matrices commute but the greatest common sublattice does not satisfy the condition given in Theorem 1. In Fig. 3(c) a pair of lattices where the interchange is possible is given. Note that, for the sake of simplicity, the members of the sets A and B in the examples are actually the angles in the exponent in (8) and (9). In each of the examples, the sets A and B show explicitly whether the interchange is possible or not.

To conclude, let us see how Theorem 1 reduces to the one-dimensional result [1]. The downsampling matrix $\boldsymbol{D}_1$ reduces to a single coefficient N_1, the upsampling matrix $\boldsymbol{D}_2$ reduces to N_2, and the greatest common sublattice of the two is actually the one corresponding to sampling by $C = \mathrm{lcm}(N_1, N_2)$. Thus $(**)$ reduces to $C = N_1 \cdot N_2$, which is equivalent to N_1 and N_2 being relatively prime. Therefore, we have the following lemma.

Lemma 1: An upsampler and a downsampler are interchangeable iff N_2 and N_1 are relatively prime, i.e.,

$$\forall k, n \in \{0 \cdots N_1 - 1\}, \qquad k \neq n \Rightarrow e^{-2\pi j (N_2/N_1) k} \neq e^{-2\pi j (N_2/N_1) n}$$

$$\Leftrightarrow$$

$$C = \mathrm{lcm}(N_1, N_2) = N_1 \cdot N_2,$$

which is exactly the statement in [1].

IV. Conclusion

We have shown how to interchange an upsampler and a downsampler in two dimensions. The result holds for arbitrary sampling lattices and is a generalization of the one-dimensional result that an upsampler commutes with a downsampler *iff* their rates are coprime.

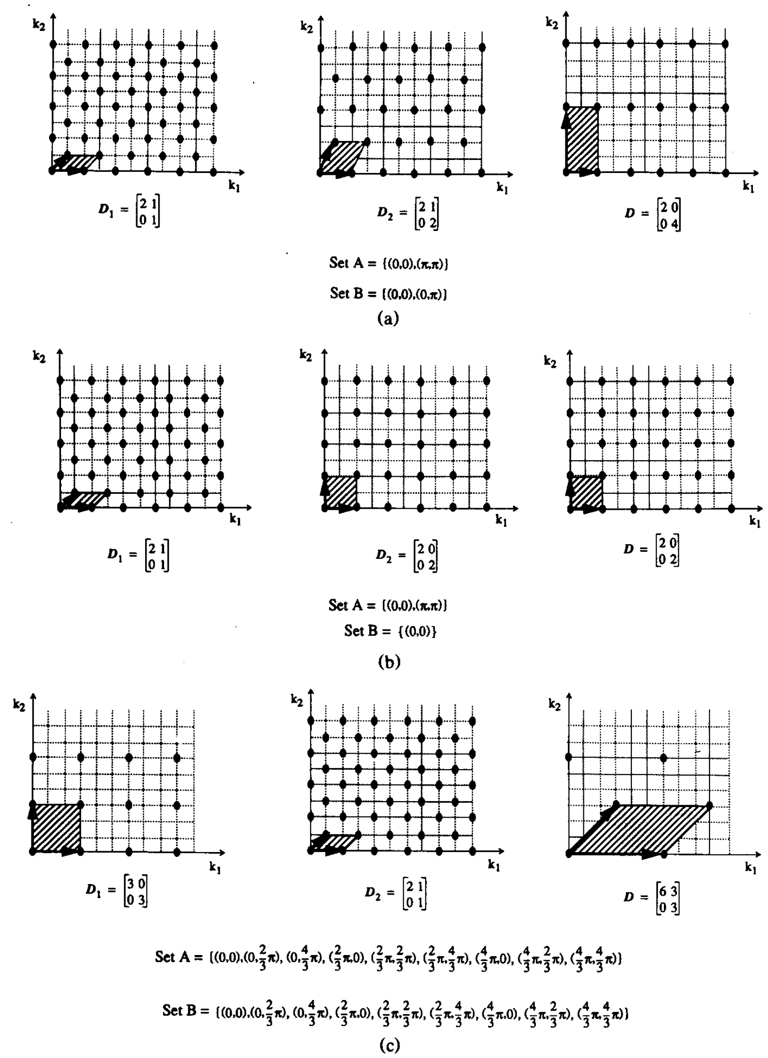

Fig. 3. (a) Downsampling lattice is "quincunx," upsampling one is "hexagonal." Matrix D_2 and inverse of D_1 do not commute and thus cannot be interchanged. (b) Downsampling lattice is "quincunx," upsampling one represents separable sampling by two in both dimensions. Now matrices commute, but greatest common sublattice of the two is the same as upsampling one and thus this pair cannot be interchanged. (c) Downsampling lattice represents separable sampling by three in both dimensions, upsampling one is "quincunx." Matrices commute and greatest common sublattice satisfies conditions of the Theorem 1. Hence, in this case, up- and downsampler can be interchanged.

Appendix A
Proof of the Theorem 1

We are going to prove the theorem for each of the three cases stated at the beginning of Section III.

A. Case $b_1 = 0 \wedge a_1 = d_1$

Since now $D_1 = a_1 I$ and D_2 is general, the matrix corresponding to the greatest common sublattice of Λ_1 and Λ_2 is of the form

$$M = \begin{pmatrix} \operatorname{lcm}(a_1, a_2) & kb_2 + la_2 \\ 0 & kd_2 \end{pmatrix}, \tag{10}$$

where k is the smallest integer such that a_1 divides kd_2 and a_1 divides $kb_2 + la_2$. Therefore $k \le a_1$ since we can always choose $k = a_1$ satisfying the above stated conditions. We are going to consider two distinct cases: when a_1 and a_2 are and are not relatively prime.

1) $\gcd(a_1, a_2) = 1$: Since a_1 and a_2 are relatively prime, the first entry in matrix M reduces to $a_1 a_2$ and

$$\det(M) = a_1 a_2 \cdot kd_2 \le a_1^2 a_2 d_2. \tag{11}$$

We can now find k from

$$kd_2 = p \operatorname{lcm}(a_1, d_2) \Rightarrow k = p \frac{a_1}{\gcd(a_1, d_2)}, \qquad p \in \mathcal{P}. \tag{12}$$

Now minimum k can be chosen as above with $p = 1$. Since a_1 and a_2 are relatively prime, a unique l can always be found such that a_1 divides $kb_2 + la_2$. Thus M is completely determined and

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$p_{n-1} + p_n = 2^{-j}$, where j is a nonnegative integer. To prove this, we only have to show that under this assumption, there exists a source distribution such that $\underline{R}_1 = R_{\text{Huff}}$. It is easy to see that

$$= H_b\left(p_{n-k+1}^{(k-1)}\right) - H_b\left(p_{n-k}^{(k)}\right) + \left[p_{n-k}^{(k-1)} + p_{n-k+1}^{(k-1)}\right]$$

(11) reduces to

$$\det(\boldsymbol{M}) = \frac{\det(\boldsymbol{D}_1)\det(\boldsymbol{D}_2)}{\gcd(a_1, d_2)}. \tag{13}$$

a) $(**) \Rightarrow (*)$: Let us suppose that $(*)$ does not hold, i.e., there exist different $\boldsymbol{k}$ and $\boldsymbol{n}$ from U_1 producing the same members of the set B, i.e.,

$$\boldsymbol{D}_2^l(\boldsymbol{D}_1^l)^{-1}\boldsymbol{m} = \boldsymbol{p} \qquad \boldsymbol{m} = \boldsymbol{k} - \boldsymbol{n}, \qquad \boldsymbol{p} \in \mathscr{Z}^2. \tag{14}$$

Since $\boldsymbol{D}_2^l$ and $(\boldsymbol{D}_1^l)^{-1}$ commute, we can rewrite this as

$$\boldsymbol{D}_2^l\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \boldsymbol{D}_1^l\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \tag{15}$$

or

$$p_1 = m_1\frac{a_2}{a_1} \qquad p_2 = m_2\frac{d_2}{a_1} + m_1\frac{b_2}{a_1}. \tag{16}$$

Since $\gcd(a_1, a_2) = 1$, $m_1 < a_1$, and p_1 is integer, it follows that m_1 has to be zero. Hence $p_2 = m_2 d_2 / a_1$. Knowing that $m_2 \neq 0$, $m_2 < a_1$ and p_2 is integer, we conclude that $\gcd(a_1, d_2) > 1$ and therefore $\det(\boldsymbol{M}) < \det(\boldsymbol{D}_1)\det(\boldsymbol{D}_2)$, which is in contradiction with our assumption.

b) $(*) \Rightarrow (**)$: Suppose that $(**)$ does not hold, i.e., $\det(\boldsymbol{M}) < \det(\boldsymbol{D}_1)\det(\boldsymbol{D}_2)$. It follows then that $\gcd(a_1, d_2) > 1$. Hence we can choose m_1 and m_2 as

$$m_1 = 0, \qquad m_2 = \frac{a_1}{\gcd(a_1, d_2)} < a_1, \tag{17}$$

yielding

$$p_1 = 0, \qquad p_2 = \frac{d_2}{\gcd(a_1, d_2)}, \tag{18}$$

again a contradiction.

2) $\gcd(a_1, a_2) > 1$: Now, obviously,

$$\det(\boldsymbol{M}) = \operatorname{lcm}(a_1, a_2)\cdot k d_2 \leq a_1 d_2 \operatorname{lcm}(a_1, a_2) < \det(\boldsymbol{D}_1)\det(\boldsymbol{D}_2), \tag{19}$$

meaning that $(**)$ is never satisfied. To prove the theorem in this case, we have to prove that $(*)$ can never be satisfied as well.

If $\gcd(a_1, d_2) > 1$, we can always choose $m_1 = 0$ and $m_2 = a_1 / \gcd(a_1, d_2) < a_1$, showing that $(*)$ does not hold.

If $\gcd(a_1, d_2) = 1$, we can always uniquely choose t_1 and t_2 such that $t_1 a_1 + t_2 a_2 = 1$. Hence the choice

$$m_1 = \frac{a_1}{\gcd(a_1, a_2)} < a_1, \qquad m_2 \equiv -m_1 b_2 t_2 (\operatorname{mod} a_1), \tag{20}$$

if $b_2 \neq 0$ or

$$m_1 = \frac{a_1}{\gcd(a_1, a_2)} < a_1, \qquad m_2 = 0 \tag{21}$$

if $b_2 = 0$, shows that there always exists a choice of m_1 and m_2 such that $(*)$ does not hold.

B. Case $b_1 = 0 \wedge b_2 = 0 \wedge a_1 \neq d_1$

Since both matrices are rectangular, $\det(\boldsymbol{D}_1) = a_1 d_1$, $\det(\boldsymbol{D}_2) = a_2 d_2$, and $\operatorname{gcs}(\Lambda_1, \Lambda_2)$ is also a rectangular lattice with $\operatorname{lcm}(a_1, a_2)$ and $\operatorname{lcm}(d_1, d_2)$ on the diagonal. Since $\operatorname{lcm}(a, b) = ab / \gcd(a, b)$, we get

$$\det(\operatorname{gcs}(\Lambda_1, \Lambda_2)) = \frac{\det(\boldsymbol{D}_1)\det(\boldsymbol{D}_2)}{\gcd(a_1, a_2)\gcd(d_1, d_2)}. \tag{22}$$

Note that this case is a separable one along the two dimensions and hence $(*)$ becomes equivalent to having $\gcd(a_1, a_2) = 1$ and $\gcd(d_1, d_2) = 1$. It is trivial to see that if the previous is true then (22) reduces to $\det(\operatorname{gcs}(\Lambda_1, \Lambda_2)) = \det(\boldsymbol{D}_1)\det(\boldsymbol{D}_2)$ satisfying $(**)$. The converse is equally easily proven for if $(**)$ holds then both $\gcd(a_1, a_2) = 1$ and $\gcd(d_1, d_2) = 1$, yielding $(*)$.

C. Case $b_1 \neq 0 \wedge d_2 = a_2 + b_2(d_1 - a_1)/b_1$

In this case we have an arbitrary nonrectangular matrix $\boldsymbol{D}_1$ and thus we can apply Proposition 1 to transform it into a rectangular matrix in $\mathscr{Z}^2$ generated by a nonstandard basis. Since commutativity is preserved, this instance reduces trivially to one of the previous two, which concludes the proof of the theorem. □

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Correction to "On Universal Hypotheses Testing Via Large Deviations"

Ofer Zeitouni and Michael Gutman

In the above paper,[1] there is a mistake in the proof of Theorem 1 (the discrete case). The proof of the Theorem as written holds true only when all coordinates of P_1 are strictly positive (i.e., when $\Sigma = \operatorname{supp} P_1$).

When some of the coordinates of P_1 are zero, it is clear that all empirical measures associated with P_1 must have $\operatorname{supp} \mu_n \subset \operatorname{supp} P_1$, and the blow up of Ω defined in (3) must take place only in the subset of Σ that belongs to $\operatorname{supp} P_1$. Thus, a priori, define $\tilde{\Sigma} = \operatorname{supp} P_1$, and restrict all probability measures to $M_1(\tilde{\Sigma})$, letting μ_n to be associated with P_2 if $\operatorname{supp} \mu_n \not\subset \tilde{\Sigma}$. The δ blow up Ω^δ will now be defined w.r.t. $M_1(\tilde{\Sigma})$, and the rest of the proof is unchanged.

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