Correspondence

Local Cosine Bases in Two Dimensions

Jelena Kovačević

***Abstract*—We construct two-dimensional (2-D) local cosine bases in discrete time. Solutions are offered both for rectangular and nonrectangular lattices. In the case of nonrectangular lattices, the problem is solved by mapping it into a one-dimensiona (1-D) equivalent problem.**

***Index Terms*—Filterbanks, local bases, wavelets.**

I. Introduction

Discrete-time cosine modulated filterbanks[1] have been in use for some time [1]–[14]. Due to a few of their properties, they have become quite popular; For example, all filters (basis functions) of a filterbank are obtained by appropriate modulation of a single prototype filter. Then, fast algorithms exist, making them very attractive for implementation. Finally, they have been recently used to achieve time-varying tilings of the time-frequency plane [5]. Their continuous-time counterpart, termed *Malvar's wavelets*, has found use in decomposing a signal into a linear combination of time-frequency atoms [6].

Local cosine bases have been used extensively in audio coding [7]–[9]. They have also found use in image coding, due to the reduction of blocking effects [10] when compared to the discrete cosine transform (DCT). Some video works contain local cosine bases as well [11]. However, in all image and video applications, one-dimensional (1-D) cosine bases are used separately. We develop here true two-dimensional (2-D) nonseparable cosine bases, which offer more degrees of freedom. We consider both rectangular and nonrectangular sampling structures, and offer solutions for both. Although in continuous time a similar analysis could be performed, here we consider the case of more interest for applications in discrete time. Note that the general solution to this problem exists (valid for any number of dimensions, arbitrary sampling lattices and both for continuous and discrete time) and is given in [12] and [13]. However, since this work was performed before [12] and [13] and also because it explicitly states the conditions for the 2-D case (especially for separable sampling which is of most interest in applications), these results should be of use to researchers and practitioners.

While revising the article, the reviewers pointed out some recent work on 2-D local cosine bases. For example, Chan in [14] considers a similar problem for separable sampling by using separable modulations and allowing the window to be nonseparable and constraining the prototype filter to have various symmetries. Work in [15] offers a general solution to the problem. Other works include [16] and [17].

Manuscript received May 27, 1995; revised October 17, 1996. Part of this work was presented at ICASSP'95, May 1995. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. C.-C. Jay Kuo.

The author is with Bell Laboratories, Innovations for Lucent Technologies, Murray Hill, NJ 07974 USA (e-mail: jelena@bell-labs.com).

Publisher Item Identifier S 1057-7149(97)07122-4.

[1]These are known under various names, such as modulated lapped transforms, Princen–Bradley filterbanks, time domain aliasing cancelation filterbanks (TDAC), modified DCT (MDCT) with 50% overlap.

II. Review of Local Cosine Bases

By local cosine bases, or modulated lapped transforms, we will denote a class of critically sampled perfect reconstruction filterbanks [9], [18] that uses a single prototype filter—window—$w[n]$ of length $2N$ (where N is the number of channels and is even) to construct all of the filters $h_0, \cdots, h_{N-1}$ as follows:

$$h_k[n] = \frac{w[n]}{\sqrt{N}} \cdot \cos\left[\frac{2k+1}{4N}(2n-N+1)\pi\right] \tag{1}$$

with $k = 0, \cdots, N-1, n = 0, \cdots, 2N-1$, and where the prototype lowpass filter $w[n]$ is symmetric, and satisfies the following condition [2], which, if we would arrange the first N coefficients of $w[n]$ along the diagonal of the matrix $\boldsymbol{W}$, could be expressed as

$$\boldsymbol{W}^2 + \boldsymbol{J}\boldsymbol{W}^2\boldsymbol{J} = 2\boldsymbol{I}. \tag{2}$$

Here, $\boldsymbol{J}$ denotes the antidiagonal matrix. This last condition, imposed on the window, ensures that the resulting local cosine basis is orthogonal. The two symmetric halves of the window are called "tails."

A convenient way of analyzing filterbanks in time domain uses infinite matrices, which describe the action of the filters on the input signal. For local cosine bases, such an infinite matrix $\boldsymbol{T}$ is "doubly diagonal" with blocks $\boldsymbol{A}_0, \boldsymbol{A}_1$, where blocks $\boldsymbol{A}_0, \boldsymbol{A}_1$ are of sizes $N \times N$, and contain the impulse responses of the filters. For example, the jth row of $\boldsymbol{A}_i$ is $(h_j[2N-1-iN] \cdots h_j[N-iN])$ for $i = 0, 1$. Note that the filter length is twice the number of channels. For an orthogonal, perfect reconstruction solution, the matrix $\boldsymbol{T}$ has to be unitary, which is equivalent to the following [19]:

$$\boldsymbol{A}_0^T\boldsymbol{A}_0 + \boldsymbol{A}_1^T\boldsymbol{A}_1 = \boldsymbol{I}, \quad \boldsymbol{A}_1^T\boldsymbol{A}_0 = \boldsymbol{A}_0^T\boldsymbol{A}_1 = \boldsymbol{0}.$$

The second set of conditions above are called the "orthogonality of tails" conditions [19]. One more fact will be of use later. Call $\boldsymbol{B}_i$ the blocks when no windowing is used, or $w_0[n] = 1, n = 0, \cdots, 2N-1$. That is, these blocks will just contain the cosines. Then

$$\boldsymbol{A}_0 = \boldsymbol{B}_0 \cdot \boldsymbol{W}, \quad \boldsymbol{A}_1 = \boldsymbol{B}_1 \cdot \boldsymbol{J}\boldsymbol{W}\boldsymbol{J} \tag{3}$$

where $\boldsymbol{W}$ is a diagonal matrix with window coefficients on the diagonal $w[0], \cdots, w[N-1]$. Blocks $\boldsymbol{B}_i$ satisfy the following:

$$\boldsymbol{B}_0^T\boldsymbol{B}_0 = \tfrac{1}{2}(\boldsymbol{I} - \boldsymbol{J}), \quad \boldsymbol{B}_1^T\boldsymbol{B}_1 = \tfrac{1}{2}(\boldsymbol{I} + \boldsymbol{J}).$$

III. 2-D Local Cosine Bases

We will now turn our attention to the 2-D case, with both rectangular and nonrectangular sampling lattices (see Fig. 1). For the current state of applications (such as image compression), rectangular sampling is of more interest; thus, we will examine it in detail and illustrate it with a design example.

A. Rectangular Sampling

We assume that we have rectangular sampling, N_1 in horizontal dimension, and N_2 in the vertical one [see Fig. 1(a)]. We will construct the filters as follows:

$$h_{ij}[n_1, n_2] = \frac{w[n_1, n_2]}{\sqrt{N_1 N_2}} \cdot \cos\left[\frac{2i+1}{4N_1}(2n_1 - N_1 + 1)\pi\right] \cdot \cos\left[\frac{2j+1}{4N_2}(2n_2 - N_2 + 1)\pi\right] \tag{4}$$

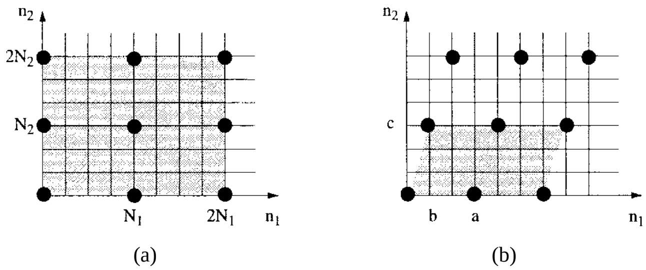

Fig. 1. Sampling lattices. The support of the filters is given by the shaded area. (a) Rectangular case. (b) Nonrectangular case.

with $i = 0, \cdots, N_1 - 1$, $j = 0, \cdots, N_2 - 1$, and $n_1 = 0, \cdots, 2N_1 - 1$, $n_2 = 0, \cdots, 2N_2 - 1$. This produces filters of sizes $2N_1 \times 2N_2$. Note that the window will be assumed to be persymmetric, that is $\boldsymbol{W} = \boldsymbol{JWJ}$.

Let us now try to find conditions for the filters to form a basis, that is, for them to be perfect reconstruction. The counterpart of one block-row of the matrix $\boldsymbol{T}$ is

$$\boldsymbol{T}_1 = (\boldsymbol{D}_0 \quad \boldsymbol{D}_1 \quad \boldsymbol{D}_2 \quad \boldsymbol{D}_3)$$

where each block $\boldsymbol{D}_i$ is of size $N_1N_2 \times N_1N_2$. Since $\boldsymbol{T}_1$ represents convolution, let us look at the shift-reversed version of one filter, h_{00}. For example, see the matrix at the bottom of the page, where dimension n_1 goes from left to right and dimension n_2 goes from bottom to top. We can arbitrarily choose a scanning order, which we define to be from left to right, and bottom to top. This means that the action of the first filter will go into $\boldsymbol{T}_1$ as the following row-vector:

$$\begin{aligned} &h_{00}[2N_1 - 1, 2N_2 - 1] \quad \cdots \quad h_{00}[0, 2N_2 - 1] \\ &\quad h_{00}[2N_1 - 1, 2N_2 - 2] \quad \cdots \quad h_{00}[2N_1 - 1, 0] \\ &\quad \cdots \quad h_{00}[0, 0]. \end{aligned}$$

Then the first row of $\boldsymbol{D}_0$, for example, will be

$$h_{00}[2N_1 - 1, 2N_2 - 1] \quad \cdots \quad h_{00}[0, 2N_2 - 1]$$

Expressing $h_{ij}[n_1, n_2]$ as in (4), we get that $\boldsymbol{D}_i$ are given by

$$\begin{aligned} \boldsymbol{D}_0 &= \boldsymbol{C}_0 \cdot \boldsymbol{W}_0, \quad & \boldsymbol{D}_2 &= \boldsymbol{C}_2 \cdot \boldsymbol{JW}_0\boldsymbol{J}, \\ \boldsymbol{D}_1 &= \boldsymbol{C}_1 \cdot \boldsymbol{W}_1, \quad & \boldsymbol{D}_3 &= \boldsymbol{C}_3 \cdot \boldsymbol{JW}_1\boldsymbol{J}. \end{aligned}$$

Here, $\boldsymbol{C}_i$ contain the modulating cosines, while diagonal matrices $\boldsymbol{W}_i$ contain appropriately placed coefficients of the 2-D, window function $w[n_1, n_2]$ as

$$\boldsymbol{W}_0 = \begin{bmatrix} w[0,0] & & \\ & \ddots & \\ & & w[N_1 - 1, N_2 - 1] \end{bmatrix},$$

$$\boldsymbol{W}_1 = \begin{bmatrix} w[N_1,0] & & \\ & \ddots & \\ & & w[2N_1 - 1, N_2 - 1] \end{bmatrix}$$

where we use the same scanning order as before. Note that since we have imposed on the window to be centrally symmetric (persymmetric), $\boldsymbol{D}_2$ and $\boldsymbol{D}_3$ have factors, $\boldsymbol{JW}_0\boldsymbol{J}$ and $\boldsymbol{JW}_1\boldsymbol{J}$, respectively. Blocks $\boldsymbol{C}_i$ are given by

$$\begin{aligned} \boldsymbol{C}_0 &= \boldsymbol{B}_{0_2} \otimes \boldsymbol{B}_{0_1}, \quad & \boldsymbol{C}_1 &= \boldsymbol{B}_{0_2} \otimes \boldsymbol{B}_{1_1} \\ \boldsymbol{C}_2 &= \boldsymbol{B}_{1_2} \otimes \boldsymbol{B}_{0_1}, \quad & \boldsymbol{C}_3 &= \boldsymbol{B}_{1_2} \otimes \boldsymbol{B}_{1_1} \end{aligned}$$

where block $\boldsymbol{B}_{i_j}$ is the ith block in dimension j as in (3). To obtain the conditions for perfect reconstruction, we have to make sure that all overlapping tails give zero, and that $\boldsymbol{T}_1$ is unitary, that is

$$\boldsymbol{D}_0^T\boldsymbol{D}_0 + \boldsymbol{D}_1^T\boldsymbol{D}_1 + \boldsymbol{D}_2^T\boldsymbol{D}_2 + \boldsymbol{D}_3^T\boldsymbol{D}_3 = \boldsymbol{I}, \tag{5}$$

$$\boldsymbol{D}_1^T\boldsymbol{D}_0 + \boldsymbol{D}_3^T\boldsymbol{D}_2 = \boldsymbol{0}, \tag{6}$$

$$\boldsymbol{D}_3^T\boldsymbol{D}_1 + \boldsymbol{D}_2^T\boldsymbol{D}_0 = \boldsymbol{0}, \tag{7}$$

$$\boldsymbol{D}_3^T\boldsymbol{D}_0 = \boldsymbol{0}, \tag{8}$$

$$\boldsymbol{D}_2^T\boldsymbol{D}_1 = \boldsymbol{0}. \tag{9}$$

Conditions (6)–(9) can be easily verified while (5) will lead to the conditions on the window, the first one being the 2-D counterpart of (2)

$$\begin{aligned} &\boldsymbol{W}_0^2 + \boldsymbol{JW}_0^2\boldsymbol{J} + \boldsymbol{W}_1^2 + \boldsymbol{JW}_1^2\boldsymbol{J} = \boldsymbol{I}, \\ &\quad -\boldsymbol{W}_0(\boldsymbol{I} \otimes \boldsymbol{J})\boldsymbol{W}_0 + \boldsymbol{JW}_0\boldsymbol{J}(\boldsymbol{I} \otimes \boldsymbol{J})\boldsymbol{JW}_0\boldsymbol{J} \\ &+\boldsymbol{W}_1(\boldsymbol{I} \otimes \boldsymbol{J})\boldsymbol{W}_1 - \boldsymbol{JW}_1\boldsymbol{J}(\boldsymbol{I} \otimes \boldsymbol{J})\boldsymbol{JW}_1\boldsymbol{J} = \boldsymbol{0}, \\ &\quad \boldsymbol{W}_0\boldsymbol{JW}_0 - \boldsymbol{W}_1\boldsymbol{JW}_1 = \boldsymbol{0}, \\ &\quad -\boldsymbol{W}_0(\boldsymbol{J} \otimes \boldsymbol{I})\boldsymbol{W}_0 + \boldsymbol{JW}_0\boldsymbol{J}(\boldsymbol{J} \otimes \boldsymbol{I})\boldsymbol{JW}_0\boldsymbol{J} \\ &-\boldsymbol{W}_1(\boldsymbol{J} \otimes \boldsymbol{I})\boldsymbol{W}_1 + \boldsymbol{JW}_1\boldsymbol{J}(\boldsymbol{J} \otimes \boldsymbol{I})\boldsymbol{JW}_1\boldsymbol{J} = \boldsymbol{0}. \end{aligned}$$

To summarize, in the 2-D case with rectangular sampling, we will have up to $(N_1N_2)/2$ free variables. Compare that to $(N_1 + N_2)/2$ free variables in the 2-D case with separable sampling.

B. Nonrectangular Sampling

The nonrectangular case is a more difficult one. We will describe a solution that will use a particular mapping from one dimension into two dimensions. Note that this solution would mean that the filters are obtained by shifting the prototype filter along a line, and that it is very similar to what was done in [20]. It will hold for an even sampling density N. First, we find the upper-triangular form of the sampling matrix

$$\boldsymbol{D} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

with $N = \det(\boldsymbol{D})$ and we assume that b and c do not have common factors [see Fig. 1(b)]. For the support of our filters, we will take two unit cells, the ones located at points [0, 0] and $[a, 0]$. Then, define the filters as follows:

$$\begin{aligned} h_k[n_1, n_2] = &\frac{w[n_1, n_2]}{\sqrt{N}} \\ &\cdot \cos\left[\frac{2k+1}{4N}(2(cn_1 - bn_2) - N + 1)\pi\right] \end{aligned} \tag{10}$$

$$\begin{array}{llllll} h_{00}[2N_1 - 1, 0] & \cdots & h_{00}[N_1, 0] & h_{00}[N_1 - 1, 0] & \cdots & h_{00}[0, 0] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{00}[2N_1 - 1, N_2 - 1] & \cdots & h_{00}[N_1, N_2 - 1] & h_{00}[N_1 - 1, N_2 - 1] & \cdots & h_{00}[0, N_2 - 1] \\ h_{00}[2N_1 - 1, N_2] & \cdots & h_{00}[N_1, N_2] & h_{00}[N_1 - 1, N_2] & \cdots & h_{00}[0, N_2] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{00}[2N_1 - 1, 2N_2 - 1] & \cdots & h_{00}[N_1, 2N_2 - 1] & h_{00}[N_1 - 1, 2N_2 - 1] & \cdots & h_{00}[0, 2N_2 - 1] \end{array}$$

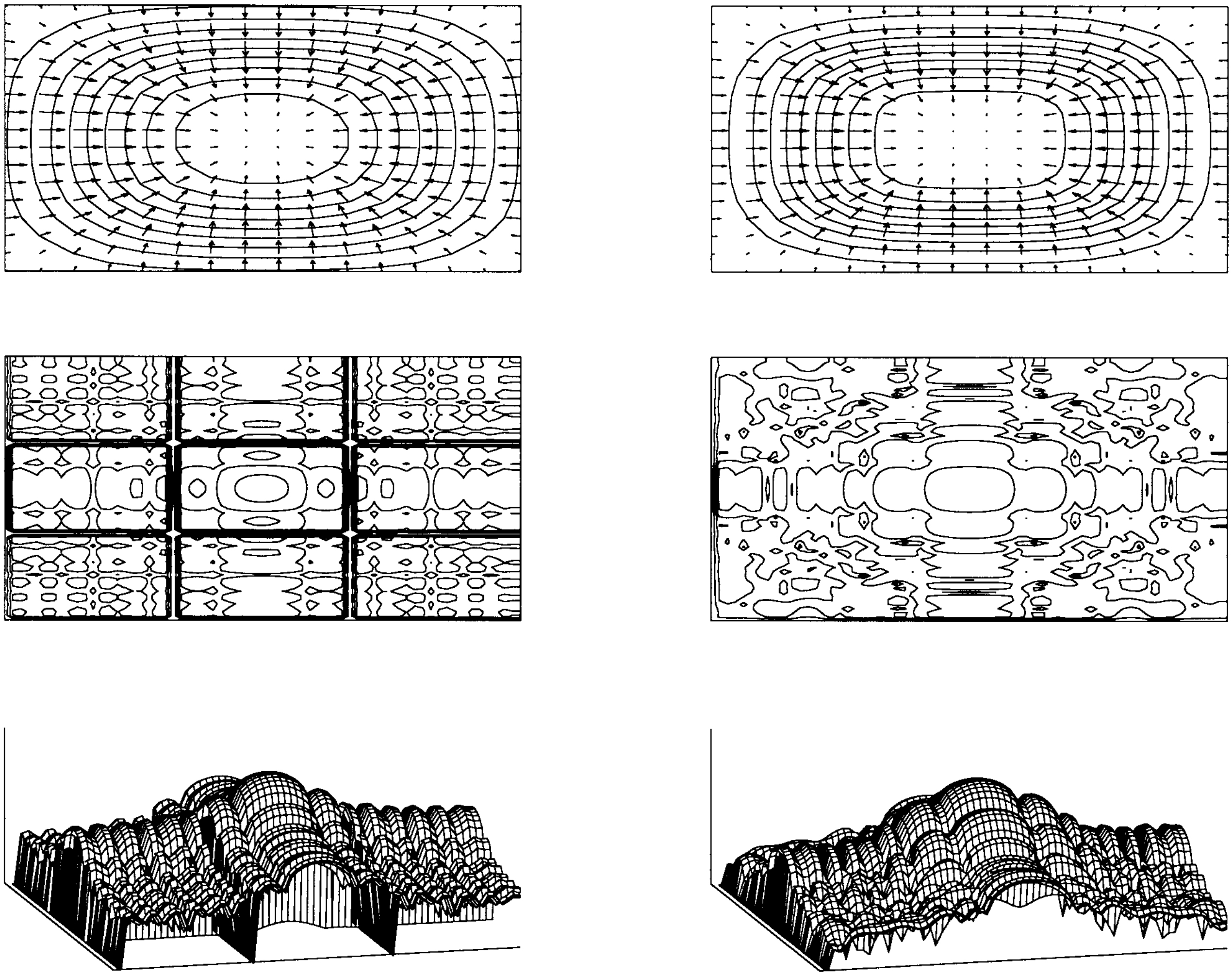

Fig. 2. Examples of the separable window (left column) versus the nonseparable window (right column) for the rectangular sampling by eight in both horizontal and vertical dimensions. The first row gives contour plots of the two windows, while the second and the third rows give contour and log plots of the magnitude of the frequency response.

with $k = 0, \cdots, N-1$, and n_1, n_2 belonging to the unit cell as explained above. Now define the scanning order to be along the second axis first, that is

$$[0,0] \quad \left[\frac{b+1}{c}, 1\right] \quad \cdots \quad \left[\frac{c-1}{c}(b+1), c-1\right] \quad [1,0] \quad \cdots . \tag{11}$$

By doing this, we have mapped the problem into a 1-D problem, that is

$$\begin{aligned} h_k[0,0] &= h_k[0] \\ h_k\left[\frac{b+1}{c}, 1\right] &= h_k[1] \\ &\vdots \\ h_k\left[2a-1+\frac{(c-1)(b+1)}{c}, c-1\right] &= h_k[2N-1]. \end{aligned}$$

Thus, if we take blocks $\boldsymbol{A}_i$ to be the formulation shown at the top of the next page, then all the relations without windowing hold and all the proofs are equivalent. We only need to take care of the window. It has to be persymmetric and

$$w^2[n_1, n_2] + w^2\left[\frac{(c-1)(b+1)}{c} + a - 1 - n, c - 1 - n_2\right] = 2$$

or equivalently

$$\boldsymbol{W}^2 + \boldsymbol{J}\boldsymbol{W}^2\boldsymbol{J} = 2\boldsymbol{I}$$

since we have used the same scanning order as in (11) to put the window coefficients into $\boldsymbol{W}$. Note that this condition is exactly the same as in the 1-D case given in (2). In the quincunx case, for example, this scheme would lead to 1-D filters. However, if we replace $cn_1 - bn_2$ with $n_1 + n_2$ where $[n_1, n_2] \in \{[0,0], [1,0], [1,1], [2,1]\}$, the whole problem is again mapped into a 1-D problem and thus easily solved. Note though, that the quincunx case is not of much interest, since it has only two channels and filters have only four taps.

C. Design Example

To illustrate constructions we presented in the previous section, we will choose rectangular subsampling and compare the separable to the nonseparable window. The sampling lattice is rectangular with sampling by eight in each dimension, thus the filters are of size 16 × 16 (as well as the window). The design procedure for the window can be found in [13]; the window was obtained by minimizing the energy in the region outside of $[-\pi/2, \pi/2] \times [-\pi/2, \pi/2]$. The result of the design procedure is given in Fig. 2. The left side shows rectangular sampling by eight in both horizontal and vertical directions with a separable window, while the right side shows the same sampling with

$$\boldsymbol{A}_0 = \begin{bmatrix} h_0\left[2a-1+\frac{c-1}{c}(b+1), c-1\right] & h_0\left[2a-1+\frac{c-2}{c}(b+1), c-2\right] & \cdots & h_0[a,0] \\ \vdots & \vdots & \vdots & \vdots \\ h_{N-1}\left[2a-1+\frac{c-1}{c}(b+1), c-1\right] & h_{N-1}\left[2a-1+\frac{c-2}{c}(b+1), c-2\right] & \cdots & h_{N-1}[a,0] \end{bmatrix}$$

$$\boldsymbol{A}_1 = \begin{bmatrix} h_0\left[a-1+\frac{c-1}{c}(b+1), c-1\right] & h_0\left[a-1+\frac{c-2}{c}(b+1), c-2\right] & \cdots & h_0[0,0] \\ \vdots & \vdots & \vdots & \vdots \\ h_{N-1}\left[a-1+\frac{c-1}{c}(b+1), c-1\right] & h_{N-1}\left[a-1+\frac{c-2}{c}(b+1), c-2\right] & \cdots & h_{N-1}[0,0] \end{bmatrix}$$

the nonseparable window. The first row gives contour plots of the two windows, while the second and the third rows give contour and log plots of the magnitude of the frequency response. The separable nature of the window in the left side is clearly seen in contour and log plots. Note that the aim in this example was not necessarily to show that one or the other filter has a superior frequency isolation. Rather, this example demonstrates that, under the same constraints, using a nonseparable window gives more freedom in design. It is clear that the separable window, being a special case of the nonseparable one, can at best equal the performance of the nonseparable window. Moreover, note how the nonseparable window demonstrates less "preference" for horizontal and vertical directions, that it is more "isotropic" as expected from a nonseparable window.

IV. Conclusions

Two-dimensional local cosine bases were presented in discrete time. We examined both rectangular and nonrectangular lattices. Although solutions for the rectangular lattices are more important in practice, those for the nonrectangular ones are a difficult challenge. In that case, the problem is solved by mapping it into an equivalent 1-D problem. As a result, solutions easily follow, however, the resulting filters will be obtained by modulation along a single line. More general modulation structures and a more general framework for local bases can solve this problem. For details, see [12] and [13].

Acknowledgment

The author thanks R. Bernardini of the University of Padua, Italy, for providing the design example and collaboration on the topic of local orthogonal bases, and T. Chen of AT&T Research for his useful comments. The author also acknowledges the anonymous reviewers who pointed out several references on 2-D local cosine bases.

References

[1] H. S. Malvar, *Signal Processing with Lapped Transforms.* Norwood, MA: Artech House, 1992.

[2] J. Princen, A. Johnson, and A. Bradley, "Subband transform coding using filter bank designs based on time domain aliasing cancellation," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Dallas, TX, Apr. 1987, pp. 2161–2164.

[3] T. A. Ramstad and J. P. Tanem, "Cosine-modulated analysis-synthesis filterbank with critical sampling and perfect reconstruction," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Toronto, Ont., Canada, May 1991, pp. 1789–1792.

[4] R. D. Koilpillai and P. P. Vaidyanathan, "New results on cosine-modulated FIR filter banks satisfying perfect reconstruction," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Toronto, Canada, May 1991, pp. 1793–1796.

[5] C. Herley, J. Kovačević, K. Ramchandran, and M. Vetterli, "Tilings of the time-frequency plane: Construction of arbitrary orthogonal bases and fast tiling algorithms," *IEEE Trans. Signal Processing,* Special Issue on Wavelets and Signal Processing, vol. 41, pp. 3341–3359, Dec. 1993.

[6] Y. Meyer, *Wavelets: Algorithms and Applications,* trans. R. D. Ryan. Philadelphia, PA: SIAM, 1993.

[7] C. C. Todd *et al.*, "AC-3: Flexible preceptual coding for audio transmission and storage," in *Proc. Conv. AES,* Amsterdam, The Netherlands, Feb. 1994.

[8] J. D. Johnston and A. J. Ferreira, "Sum-difference stereo transform coding," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* San Francisco, CA, Mar. 1992, pp. II:569–572.

[9] M. Vetterli and J. Kovačević, *Wavelets and Subband Coding, Signal Processing.* Englewood Cliffs, NJ: Prentice-Hall, 1995.

[10] J. Kovačević, D. J. LeGall, and M. Vetterli, "Image coding with windowed modulated filter banks," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Glasgow, U.K., May 1989, pp. 1949–1952.

[11] A. W. Johnson, J. Princen, and M. H. Chan, "Frequency scalable video coding using the MDCT," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Adelaide, Australia, 1994.

[12] R. Bernardini and J. Kovačević, "Local orthogonal bases I: Construction," *Multidimen. Syst. Signal Process.*, Special Issue Wavelets Multires. Signal Process., vol. 7, no. 4, July 1996, invited paper.

[13] R. Bernardini and J. Kovačević, "Local orthogonal bases II: Window design," *Multidimen. Syst. Signal Process.,* Special Issue Wavelets Multires. Signal Process., vol. 7, no. 4, July 1996.

[14] S. C. Chan, "Two dimensional nonseparable modulated filter banks," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Adelaide, Australia, 1994.

[15] Y.-P. Lin and P. P. Vaidyanathan, "Two-dimensional paraunitary cosine modulated perfect reconstruction filter banks," in *Proc. IEEE Int. Symp. Circuits and Systems,* Seattle, WA, 1995.

[16] M. Ikehara, "Cosine-modulated 2-dimensional filter banks satisfying perfect reconstruction," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing,* Adelaide, Australia, 1994.

[17] H. Yan and M. Ikehara, "Modulated 2-dimensional perfect reconstruction FIR filter banks with permissible subbands," in *Proc. IEEE Int. Symp. Circuits and Systems,* Seattle, WA, 1995.

[18] P. P. Vaidyanathan, *Multirate Systems and Fitler Banks.* Englewood Cliffs, NJ: Prentice-Hall, 1992.

[19] M. Vetterli and D. J. LeGall, "Perfect reconstruction FIR filter banks: Some properties and factorizations," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. 37, pp. 1057–1071, July 1989.

[20] A. Tabatabai, "Some results on two-dimensional pseudoquadrature mirror filters," *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 988–992, Aug. 1987.