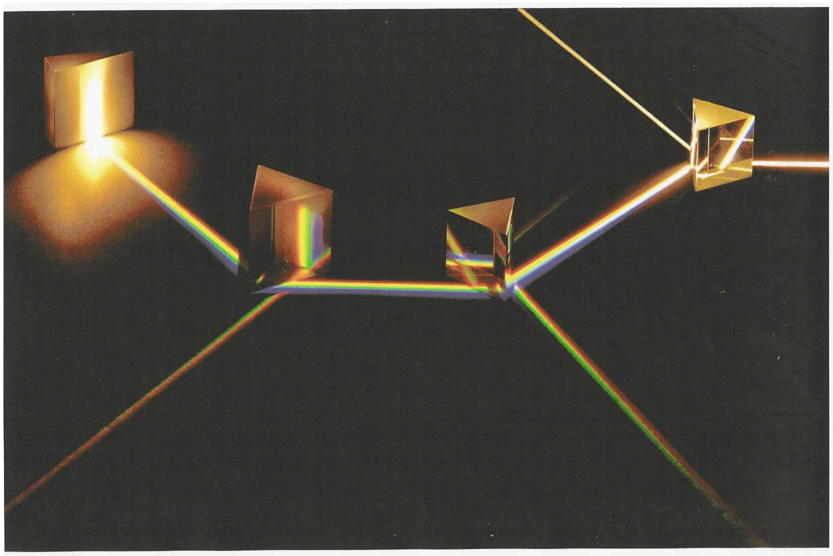
Foundations of Signal Processing
Tables



Martin Vetterli
École Polytechnique Fédérale de Lausanne and University of California, Berkeley

Jelena Kovačević
Carnegie Mellon University

Vivek K Goyal
Massachusetts Institute of Technology

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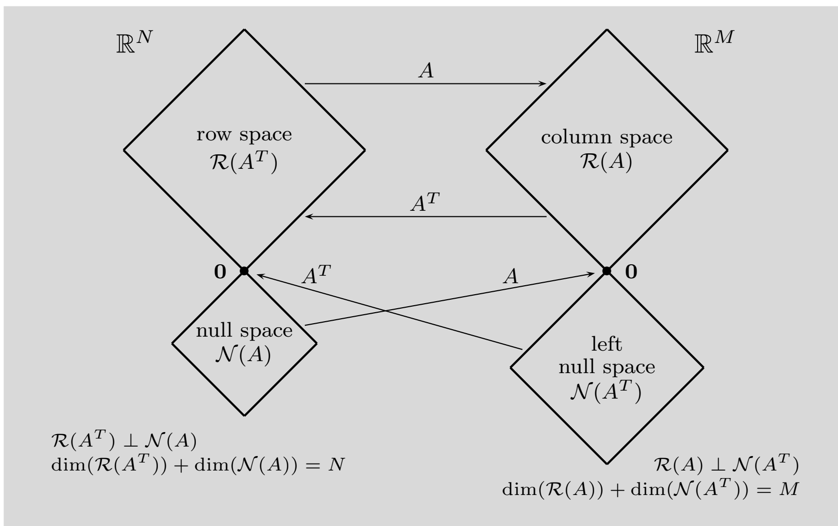
Cover photograph by Christiane Grimm, Geneva, Switzerland.

Experimental set up by Prof. Libero Zuppiroli and Philippe Bugnon, Laboratory of Optoelectronics Molecular Materials, EPFL, Lausanne, Switzerland.

The photograph captures an experiment first described by Isaac Newton in *Opticks* in 1730, explaining how the white light can be split into its color components and then resynthesized. It is a physical implementation of a white light decomposition into its Fourier components—the colors of the rainbow, followed by a synthesis to recover the original. This experiment graphically summarizes the major theme of the book—many signals can be split into essential components, and the signal's characteristics can be better understood by looking at its components; the process called *analysis*. These components can be used for processing the signal; more importantly, the signal can be perfectly recovered from those same components, through the process called *synthesis*.

Fundamental subspaces associated with matrix A

The four fundamental subspaces associated with a real matrix $A \in \mathbb{R}^{M \times N}$. The matrix determines an orthogonal decomposition of $\mathbb{R}^N$ into the row space of A and the null space of A; and an orthogonal decomposition of $\mathbb{R}^M$ into the column space (range) of A and the left null space of A. The column and row spaces of A have the same dimension, which equals the rank of A. (Figure inspired by the cover of G. Strang. *Introduction to Linear Algebra*. http://math.mit.edu/linearalgebra/, 4th edition, 2009.)



Space	**Symbol**	**Definition**	**Dimension**
Column space (range)	$\mathcal{R}(A)$	$\{y \in \mathbb{C}^M \mid y = Ax \text{ for some } x \in \mathbb{C}^N\}$	$\text{rank}(A)$
Left null space	$\mathcal{N}(A^*)$	$\{y \in \mathbb{C}^M \mid A^*y = 0\}$	$M - \text{rank}(A)$
	$\dim(\mathcal{R}(A)) + \dim(\mathcal{N}(A^*)) = M$		
Row space	$\mathcal{R}(A^*)$	$\{x \in \mathbb{C}^N \mid x = A^*y \text{ for some } y \in \mathbb{C}^M\}$	$\text{rank}(A)$
Null space (kernel)	$\mathcal{N}(A)$	$\{x \in \mathbb{C}^N \mid Ax = 0\}$	$N - \text{rank}(A)$
	$\dim(\mathcal{R}(A^*)) + \dim(\mathcal{N}(A)) = N$		

Discrete-time processing concepts

Concept	Notation	Infinite-length sequences	Finite-length sequences
Shift	δ_{n-1}	linear	circular
Sequence vector	x_n	$n \in \mathbb{Z}$	$n \in \{0, 1, \ldots, N-1\}$
LSI system filter, impulse response operator	h_n	$n \in \mathbb{Z}$	$n \in \{0, 1, \ldots, N-1\}$
Convolution	$h * x$	$\sum_{k \in \mathbb{Z}} x_k h_{n-k}$	$\sum_{k=0}^{N-1} x_k h_{N,(n-k) \bmod N}$
Eigensequence satisfies invariant space	v $h * v_\lambda = \lambda v_\lambda$ S_λ	$e^{j\omega n}$ $h * v_\omega = H(e^{j\omega})\, v_\omega$ $S_\omega = \{\alpha\, e^{j\omega n}\}$ $\alpha \in \mathbb{C}, \omega \in \mathbb{R}$	$e^{j(2\pi/N)kn}$ $h * v_k = H_k\, v_k$ $S_k = \{\alpha\, e^{j(2\pi/N)kn}\}$ $\alpha \in \mathbb{C}, k \in \mathbb{Z}$
Frequency response eigenvalue	λ	$\lambda_\omega = H(e^{j\omega})$ $\sum_{n \in \mathbb{Z}} h_n e^{-j\omega n}$	$\lambda_k = H_k$ $\sum_{n=0}^{N-1} h_n e^{-j(2\pi/N)kn}$
Fourier transform spectrum	X	DTFT $X(e^{j\omega}) = \sum_{n \in \mathbb{Z}} x_n e^{-j\omega n}$	DFT $X_k = \sum_{n=0}^{N-1} x_n e^{-j(2\pi/N)kn}$

Multirate discrete-time processing concepts

Concept	Expression
Sampling factor 2	
Input	$x_n, X(z), X(e^{j\omega})$
Downsampling by 2	$y_n = x_{2n}$ $y = D_2\, x$ $Y(z) = (1/2)\left[X(z^{1/2}) + X(-z^{1/2})\right]$ $Y(e^{j\omega}) = (1/2)\left[X(e^{j\omega/2}) + X(e^{j(\omega-2\pi)/2})\right]$
Upsampling by 2	$y_n = \begin{cases} x_{n/2}, & n \text{ even;} \\ 0, & \text{otherwise.} \end{cases}$ $y = U_2\, x$ $Y(z) = X(z^2)$ $Y(e^{j\omega}) = X(e^{j2\omega})$
Filtering with h & downsampling by 2	$y = D_2 H\, x$ $Y(z) = (1/2)\frac{1}{2}\left[H(z^{1/2})X(z^{1/2}) + H(-z^{1/2})X(-z^{1/2})\right]$ $Y(e^{j\omega}) = (1/2)\left[H(e^{j\omega/2})X(e^{j\omega/2}) + H(e^{j(\omega-2\pi)/2})X(e^{j(\omega-2\pi)/2})\right]$
Upsampling by 2 & filtering with g	$y = GU_2\, x$ $Y(z) = G(z)X(z^2)$ $Y(e^{j\omega}) = G(e^{j\omega})X(e^{j2\omega})$
Sampling factor N	
Input	$x_n, X(z), X(e^{j\omega})$
Downsampling by N	$y_n = x_{Nn}$ $y = D_N\, x$ $Y(z) = (1/N)\sum_{k=0}^{N-1} X(\omega_N^k z^{1/N})$ $Y(e^{j\omega}) = (1/N)\sum_{k=0}^{N-1} X(e^{j(\omega-2\pi k/N)})$
Upsampling by N	$y_n = \begin{cases} x_{n/N}, & n/N \in \mathbb{Z}; \\ 0, & \text{otherwise.} \end{cases}$ $y = U_N\, x$ $Y(z) = X(z^N)$ $Y(e^{j\omega}) = X(e^{jN\omega})$

Kronecker delta sequence properties

Kronecker delta sequence	
Normalization	$\sum_{n\in\mathbb{Z}} \delta_n = 1$
Sifting	$\sum_{n\in\mathbb{Z}} x_{n_0-n}\delta_n = \sum_{n\in\mathbb{Z}} \delta_{n_0-n}x_n \;=\; x_{n_0}$
Shifting	$x_n *_n \delta_{n-n_0} = x_{n-n_0}$
Sampling	$x_n\,\delta_n = x_0\,\delta_n$
Restriction	$x_n\,\delta_n = 1_{\{0\}}x$

Dirac delta function properties

Dirac delta function	
Normalization	$\int_{-\infty}^{\infty} \delta(t)\ dt = 1$
Sifting	$\int_{-\infty}^{\infty} x(t_0-t)\ \delta(t)\ dt = \int_{-\infty}^{\infty} x(t)\ \delta(t_0-t)\ dt = x(t_0)$
Shifting	$x(t) *_t \delta(t-t_0) = x(t-t_0)$
Sampling	$x(t)\,\delta(t) = x(0)\,\delta(t)$
Scaling	$\delta(t/a) = \lvert a\rvert\,\delta(t)$ for any nonzero $a \in \mathbb{R}$

Ideal filters with unit-norm impulse response

Ideal filters	Time domain	DTFT domain
Ideal lowpass filter	$\sqrt{\frac{\omega_0}{2\pi}}\,\mathrm{sinc}(\omega_0 n/2)$	$\begin{cases} \sqrt{2\pi/\omega_0}, & \|\omega\| \le \omega_0/2; \\ 0, & \text{otherwise.} \end{cases}$
Ideal Nth-band filter	$(1/\sqrt{N})\,\mathrm{sinc}(\pi n/N)$	$\begin{cases} \sqrt{N}, & \|\omega\| \le \pi/N; \\ 0, & \text{otherwise.} \end{cases}$
Ideal halfband lowpass filter	$(1/\sqrt{2})\,\mathrm{sinc}(\pi n/2)$	$\begin{cases} \sqrt{2}, & \|\omega\| \le \pi/2; \\ 0, & \text{otherwise.} \end{cases}$

Unit-norm box and sinc functions/sequences

Functions on the real line		**FT**
	$x(t), \quad t \in \mathbb{R}, \quad \\|x\\| = 1$	$X(\omega), \quad \omega \in \mathbb{R}, \quad \\|X\\| = \sqrt{2\pi}$
Box	$\begin{cases} 1/\sqrt{t_0}, & \|t\| \le t_0/2; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{t_0}\,\mathrm{sinc}\,(\omega t_0/2)$
Sinc	$\sqrt{\frac{\omega_0}{2\pi}}\,\mathrm{sinc}(\omega_0 t/2)$	$\begin{cases} \sqrt{2\pi/\omega_0}, & \|\omega\| \le \omega_0/2; \\ 0, & \text{otherwise.} \end{cases}$
Periodic functions		**FS**
	$x(t), \quad t \in [-T/2, T/2), \quad \\|x\\| = 1$	$X_k, \quad k \in \mathbb{Z}, \quad \\|X\\| = 1/\sqrt{T}$
Box	$\begin{cases} 1/\sqrt{t_0}, & \|t\| \le t_0/2; \\ 0, & \text{otherwise.} \end{cases}$	$\frac{\sqrt{t_0}}{T}\,\mathrm{sinc}\,(\pi k t_0/T)$
Sinc	$\sqrt{\frac{k_0}{T}}\,\frac{\mathrm{sinc}(\pi k_0 t/T)}{\mathrm{sinc}(\pi t/T)}$	$\begin{cases} 1/\sqrt{k_0 T}, & \|k\| \le (k_0-1)/2; \\ 0, & \text{otherwise.} \end{cases}$
Infinite-length sequences		**DTFT**
	$x_n, \quad n \in \mathbb{Z}, \quad \\|x\\| = 1$	$X(e^{j\omega}), \quad \omega \in [-\pi, \pi), \quad \\|X\\| = \sqrt{2\pi}$
Box	$\begin{cases} 1/\sqrt{n_0}, & \|n\| \le (n_0-1)/2; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{n_0}\,\frac{\mathrm{sinc}\,(\omega n_0/2)}{\mathrm{sinc}\,(\omega/2)}$
Sinc	$\sqrt{\frac{\omega_0}{2\pi}}\,\mathrm{sinc}(\omega_0 n/2)$	$\begin{cases} \sqrt{2\pi/\omega_0}, & \|\omega\| \le \omega_0/2; \\ 0, & \text{otherwise.} \end{cases}$
Finite-length sequences		**DFT**
	$x_n, \quad n \in \{0, 1, \ldots, N-1\}, \quad \\|x\\| = 1$	$X_k, \quad k \in \{0, 1, \ldots, N-1\}, \quad \\|X\\| = \sqrt{N}$
Box	$\begin{cases} 1/\sqrt{n_0}, & \|n - N/2\| \ge (n_0-1)/2; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{n_0}\,\frac{\mathrm{sinc}\,(\pi n_0 k/N)}{\mathrm{sinc}\,(\pi k/N)}$
Sinc	$\sqrt{\frac{k_0}{N}}\,\frac{\mathrm{sinc}\,(\pi n k_0/N)}{\mathrm{sinc}\,(\pi n/N)}$	$\begin{cases} \sqrt{N/k_0}, & \|k - N/2\| \ge (k_0-1)/2; \\ 0, & \text{otherwise.} \end{cases}$

Energy and power concepts for sequences

Deterministic sequences	WSS discrete-time stochastic processes
Energy spectral density	Power spectral density
$A(e^{j\omega}) \;=\; \lvert X(e^{j\omega})\rvert^2$	$A(e^{j\omega}) \;=\; \sum_{k\in\mathbb{Z}} \mathrm{E}[\, \mathrm{x}_n\, \mathrm{x}_{n-k}^* \,]\, e^{-j\omega k}$
Energy	Power
$E \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} A(e^{j\omega})\, d\omega$	$P \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} A(e^{j\omega})\, d\omega$
$E \;=\; a_0 \;=\; \sum_{n\in\mathbb{Z}} \lvert x_n\rvert^2$	$P \;=\; a_0 \;=\; \mathrm{E}[\, \lvert \mathrm{x}_n\rvert^2 \,]$

Orthogonality concepts for sequences

	Deterministic sequences	WSS discrete-time stochastic processes
Time	$c_{x,y,k} = \langle x_n,\, y_{n-k}\rangle_n = 0$	$c_{\mathrm{x},\mathrm{y},k} = \mathrm{E}[\, \mathrm{x}_n \mathrm{y}_{n-k}^* \,] = 0$
Frequency	$C_{x,y}(e^{j\omega}) = X(e^{j\omega})\, Y^*(e^{j\omega}) = 0$	$C_{\mathrm{x},\mathrm{y}}(e^{j\omega}) = 0$

Deterministic discrete-time autocorrelation/crosscorrelation properties

Domain	Autocorrelation/Crosscorrelation		Properties
Sequences		$x_n,\, y_n$	
Time	a_n	$\sum_{k\in\mathbb{Z}} x_k\, x^*_{k-n}$	$a_n = a^*_{-n}$
	c_n	$\sum_{k\in\mathbb{Z}} x_k\, y^*_{k-n}$	$c_{x,y,n} = c^*_{y,x,-n}$
DTFT	$A(e^{j\omega})$	$\lvert X(e^{j\omega})\rvert^2$	$A(e^{j\omega}) = A^*(e^{j\omega})$
	$C(e^{j\omega})$	$X(e^{j\omega})\, Y^*(e^{j\omega})$	$C_{x,y}(e^{j\omega}) = C^*_{y,x}(e^{j\omega})$
z-transform	$A(z)$	$X(z)\, X_*(z^{-1})$	$A(z) = A_*(z^{-1})$
	$C(z)$	$X(z)\, Y_*(z^{-1})$	$C_{x,y}(z) = C_{y,x*}(z^{-1})$
DFT	A_k	$\lvert X_k\rvert^2$	$A_k = A^*_{-k \bmod N}$
	C_k	$X_k\, Y^*_k$	$C_k = C^*_{y,x,-k \bmod N}$
Real sequences		$x_n,\, y_n$	
Time	a_n	$\sum_{k\in\mathbb{Z}} x_k\, x_{k-n}$	$a_n = a_{-n}$
	c_n	$\sum_{k\in\mathbb{Z}} x_k\, y_{k-n}$	$c_{x,y,n} = c_{y,x,-n}$
DTFT	$A(e^{j\omega})$	$\lvert X(e^{j\omega})\rvert^2$	$A(e^{j\omega}) = A(e^{-j\omega})$
	$C(e^{j\omega})$	$X(e^{j\omega})\, Y(e^{-j\omega})$	$C_{x,y}(e^{j\omega}) = C_{y,x}(e^{-j\omega})$
z-transform	$A(z)$	$X(z)\, X(z^{-1})$	$A(z) = A(z^{-1})$
	$C(z)$	$X(z)\, Y(z^{-1})$	$C_{x,y}(z) = C_{y,x}(z^{-1})$
DFT	A_k	$\lvert X_k\rvert^2$	$A_k = A_{-k \bmod N}$
	C_k	$X_k\, Y_k$	$C_k = C_{y,x,-k \bmod N}$
Vector of sequences		$\begin{bmatrix} x_{0,n} & x_{1,n} \end{bmatrix}^T$	
Time	A_n	$\begin{bmatrix} a_{0,n} & c_{0,1,n} \\ c_{1,0,n} & a_{1,n} \end{bmatrix}$	$A_n = A^*_{-n}$ $A_n = A^T_{-n}$
DTFT	$A(e^{j\omega})$	$\begin{bmatrix} A_0(e^{j\omega}) & C_{0,1}(e^{j\omega}) \\ C_{1,0}(e^{j\omega}) & A_1(e^{j\omega}) \end{bmatrix}$	$A(e^{j\omega}) = A^*(e^{j\omega})$ $A(e^{j\omega}) = A^T(e^{-j\omega})$
z-transform	$A(z)$	$\begin{bmatrix} A_0(z) & C_{0,1}(z) \\ C_{1,0}(z) & A_1(z) \end{bmatrix}$	$A(z) = A_*(z^{-1})$ $A(z) = A^T(z^{-1})$
DFT	A_k	$\begin{bmatrix} A_{0,k} & C_{0,1,k} \\ C_{1,0,k} & A_{1,k} \end{bmatrix}$	$A_k = A^*_{-k \bmod N}$ $A_k = A^T_{-k \bmod N}$

Discrete/continuous Fourier transform summary

Transform	Forward/Inverse	Duality/Periodicity
Fourier transform	$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$	
Fourier series	$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j(2\pi/T)kt}\, dt$ $x(t) = \sum_{k\in\mathbb{Z}} X_k\, e^{j(2\pi/T)kt}$	dual with DTFT $x(t+T) = x(t)$
Discrete-time Fourier transform	$X(e^{j\omega}) = \sum_{n\in\mathbb{Z}} x_n\, e^{-j\omega n}$ $x_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$	dual with Fourier series $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$
Discrete Fourier transform	$X_k = \sum_{n=0}^{N-1} x_n\, e^{-j(2\pi/N)kn}$ $x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k\, e^{j(2\pi/N)kn}$	

Discrete Fourier transform (DFT) properties

DFT properties	Time domain	DFT domain
Basic properties		
Linearity	$\alpha x_n + \beta y_n$	$\alpha X_k + \beta Y_k$
Circular shift in time	$x_{(n-n_0) \bmod N}$	$W_N^{kn_0} X_k$
Circular shift in frequency	$W_N^{-k_0 n} x_n$	$X_{(k-k_0) \bmod N}$
Circular time reversal	$x_{-n \bmod N}$	$X_{-k \bmod N}$
Circular convolution in time	$(h \circledast x)_n$	$H_k\, X_k$
Circular convolution in frequency	$h_n\, x_n$	$\frac{1}{N}(H \circledast X)_k$
Circular deterministic autocorrelation	$a_n = \sum_{k=0}^{N-1} x_k\, x^*_{(k-n) \bmod N}$	$A_k = \vert X_k\vert^2$
Circular deterministic crosscorrelation	$c_n = \sum_{k=0}^{N-1} x_k\, y^*_{(k-n) \bmod N}$	$C_k = X_k\, Y_k^*$
Parseval's equality	$\Vert x\Vert^2 = \sum_{n=0}^{N-1} \vert x_n\vert^2 = \frac{1}{N}\sum_{k=0}^{N-1} \vert X_k\vert^2 = \frac{1}{N}\Vert X\Vert^2$	
Related sequences		
Conjugate	x_n^*	$X^*_{-k \bmod N}$
Conjugate, time reversed	$x^*_{-n \bmod N}$	X_k^*
Real part	$\Re(x_n)$	$(X_k + X^*_{-k \bmod N})/2$
Imaginary part	$\Im(x_n)$	$(X_k - X^*_{-k \bmod N})/(2j)$
Conjugate-symmetric part	$(x_n + x^*_{-n \bmod N})/2$	$\Re(X_k)$
Conjugate-antisymmetric part	$(x_n - x^*_{-n \bmod N})/(2j)$	$\Im(X_k)$
Symmetries for real x		
X conjugate symmetric		$X_k = X^*_{-k \bmod N}$
Real part of X even		$\Re(X_k) = \Re(X_{-k \bmod N})$
Imaginary part of X odd		$\Im(X_k) = -\Im(X_{-k \bmod N})$
Magnitude of X even		$\vert X_k\vert = \vert X_{-k \bmod N}\vert$
Phase of X odd		$\arg X_k = -\arg X_{-k \bmod N}$
Common transform pairs		
Kronecker delta sequence	δ_n	1
Shifted Kronecker delta sequence	$\delta_{(n-n_0) \bmod N}$	$W_N^{kn_0}$
Constant sequence	1	$N\delta_k$s
Geometric sequence	α^n	$(1-\alpha W_N^{kN})/(1-\alpha W_N^k)$
Periodic sinc sequence (ideal lowpass filter)	$\sqrt{\frac{k_0}{N}}\,\frac{\operatorname{sinc}(\pi n k_0/N)}{\operatorname{sinc}(\pi n/N)}$	$\begin{cases} \sqrt{\frac{N}{k_0}}, & \left\vert k-\frac{N}{2}\right\vert \geq \frac{k_0-1}{2}; \\ 0, & \text{otherwise.} \end{cases}$
Box sequence	$\begin{cases} \frac{1}{\sqrt{n_0}}, & \left\vert n-\frac{N}{2}\right\vert \geq \frac{n_0-1}{2}; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{n_0}\,\frac{\operatorname{sinc}(\pi n_0 k/N)}{\operatorname{sinc}(\pi k/N)}$

Fourier series properties

FS properties	Time domain	FS domain
Basic properties		
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X_k + \beta Y_k$
Shift in time	$x(t - t_0)$	$e^{-j(2\pi/T)kt_0}\, X_k$
Shift in frequency	$e^{j(2\pi/T)k_0 t}\, x(t)$	$X(k - k_0)$
Time reversal	$x(-t)$	X_{-k}
Differentiation	$d^n x(t)/dt^n$	$(j2\pi k/T)^n X_k$
Integration	$\int_{-T/2}^{t} x(\tau)\, d\tau$	$(T/(j2\pi k))\, X_k,\ X_0 = 0$
Circular convolution in time	$(h \circledast x)(t)$	$T\, H_k\, X_k$
Convolution in frequency	$h(t)\, x(t)$	$(H * X)_k$
Circular deterministic autocorrelation	$a(t) = \int_{-T/2}^{T/2} x(\tau)\, x^*(\tau - t)\, d\tau$	$A_k = T\, \lvert X_k\rvert^2$
Circular deterministic crosscorrelation	$c(t) = \int_{-T/2}^{T/2} x(\tau)\, y^*(\tau - t)\, d\tau$	$C_k = T\, X_k Y_k^*$
Parseval's equality	$\lVert x\rVert^2 = \int_{-T/2}^{T/2} \lvert x(t)\rvert^2\, dt = T \sum_{k\in\mathbb{Z}} \lvert X_k\rvert^2 = T\, \lVert X\rVert^2$	
Related functions		
Conjugate	$x^*(t)$	X^*_{-k}
Conjugate, time reversed	$x^*(-t)$	X^*_k
Real part	$\Re(x(t))$	$(X_k + X^*_{-k})/2$
Imaginary part	$\Im(x(t))$	$(X_k - X^*_{-k})/(2j)$
Conjugate-symmetric part	$(x(t) + x^*(-t))/2$	$\Re(X_k)$
Conjugate-antisymmetric part	$(x(t) - x^*(-t))/(2j)$	$\Im(X_k)$
Symmetries for real x		
X conjugate symmetric		$X_k = X^*_{-k}$
Real part of X even		$\Re(X_k) = \Re(X_{-k})$
Imaginary part of X odd		$\Im(X_k) = -\Im(X_{-k})$
Magnitude of X even		$\lvert X_k\rvert = \lvert X_{-k}\rvert$
Phase of X odd		$\arg X_k = -\arg X_{-k}$
Common transform pairs		
Dirac comb	$\sum_{n\in\mathbb{Z}} \delta(t - nT)$	$1/T$
Periodic sinc function (ideal lowpass filter)	$\sqrt{\frac{k_0}{T}}\, \frac{\mathrm{sinc}(\pi k_0 t/T)}{\mathrm{sinc}(\pi t/T)}$	$\begin{cases} 1/\sqrt{k_0 T}, & \lvert k\rvert \le (k_0 - 1)/2; \\ 0, & \text{otherwise.} \end{cases}$
Box function (one period)	$\begin{cases} 1/\sqrt{t_0}, & \lvert t\rvert \le t_0/2; \\ 0, & t_0/2 < \lvert t\rvert \le T/2. \end{cases}$	$\frac{\sqrt{t_0}}{T}\, \mathrm{sinc}\,(\pi k t_0/T)$
Square wave (one period with $T = 1$)	$\begin{cases} -1, & t \in [-1/2, 0); \\ 1, & t \in [0, 1/2). \end{cases}$	$\begin{cases} 2j/(\pi k), & k \text{ odd}; \\ 0, & k \text{ even}. \end{cases}$
Triangle wave (one period with $T = 1$)	$\frac{1}{2} - \lvert t\rvert, \quad \lvert t\rvert \le 1/2.$	$\begin{cases} 1/4, & k = 0; \\ 1/(\pi k)^2, & k \text{ odd}; \\ 0, & k \neq 0 \text{ even}. \end{cases}$
Sawtooth wave (one period with $T = 1$)	$2t, \quad \lvert t\rvert \le 1/2.$	$\begin{cases} 0, & k = 0; \\ j(-1)^k/(\pi k), & k \neq 0. \end{cases}$

z-transform properties

z-transform properties	Time domain	z domain	ROC
ROC properties	General seq.		$0 \le r_1 < \lvert z\rvert < r_2 \le \infty$
	Finite-length seq.		all z, except possibly $0, \infty$
	Right-sided seq.		$\lvert z\rvert >$ largest pole
	Left-sided seq.		$\lvert z\rvert <$ smallest pole
	BIBO stable		$\supset \lvert z\rvert = 1$
Basic properties			
Linearity	$\alpha x_n + \beta y_n$	$\alpha X(z) + \beta Y(z)$	$\supset \text{ROC}_x \cap \text{ROC}_y$
Shift in time	x_{n-n_0}	$z^{-n_0} X(z)$	ROC_x
Scaling in time			
Downsampling	x_{Nn}	$\frac{1}{N}\sum_{k=0}^{N-1} X(W_N^k z^{1/N})$	$(\text{ROC}_x)^{1/N}$
Upsampling	$\begin{cases} x_{n/N}, & n/N \in \mathbb{Z}; \\ 0, & \text{otherwise.} \end{cases}$	$X(z^N)$	$(\text{ROC}_x)^N$
Scaling in z	$\alpha^n x_n$	$X(\alpha^{-1} z)$	$\lvert\alpha\rvert\, \text{ROC}_x$
Time reversal	x_{-n}	$X(z^{-1})$	$1/\text{ROC}_x$
Differentiation	$n^k x_n$	$(-1)^k z^k \partial^k X(z)/\partial z^k$	ROC_x
	$m_k = \sum_{n\in\mathbb{Z}} n^k x_n = (-1)^k \partial^k X(z)/\partial z^k \big\rvert_{z=1}$		
Convolution in time	$(h * x)_n$	$H(z)\, X(z)$	$\supset \text{ROC}_h \cap \text{ROC}_x$
Deterministic autocorrelation	$a_n = \sum_{k\in\mathbb{Z}} x_k\, x_{k-n}^*$	$A(z) = X(z)\, X_*(z^{-1})$	$\text{ROC}_x \cap 1/\text{ROC}_x$
Deterministic crosscorrelation	$c_n = \sum_{k\in\mathbb{Z}} x_k\, y_{k-n}^*$	$C(z) = X(z)\, Y_*(z^{-1})$	$1/\text{ROC}_x \cap \text{ROC}_y$
Related sequences			
Conjugate	x_n^*	$X^*(z^*)$	ROC_x
Conjugate, time reversed	x_{-n}^*	$X_*(z^{-1})$	$1/\text{ROC}_x$
Real part	$\Re(x_n)$	$(X(z) + X^*(z^*))/2$	ROC_x
Imaginary part	$\Im(x_n)$	$(X(z) - X^*(z^*))/(2j)$	ROC_x
Symmetries for real x			
X conjugate symmetric		$X(z) = X^*(z^*)$	
Common transform pairs			
Kronecker delta sequence	δ_n	1	all z
Shifted Kronecker delta sequence	δ_{n-n_0}	z^{-n_0}	all z, except possibly $0, \infty$
Geometric sequence	$\alpha^n u_n$	$1/(1 - \alpha z^{-1})$	$\lvert z\rvert > \lvert\alpha\rvert$
	$-\alpha^n u_{-n-1}$		$\lvert z\rvert < \lvert\alpha\rvert$
Arithmetic-geometric sequence	$n\, \alpha^n u_n$	$(\alpha z^{-1})/(1 - \alpha z^{-1})^2$	$\lvert z\rvert > \lvert\alpha\rvert$
	$-n\, \alpha^n u_{-n-1}$		$\lvert z\rvert < \lvert\alpha\rvert$

DTFT: Discrete-time Fourier transform properties

DTFT properties	Time domain	DTFT domain
Basic properties		
Linearity	$\alpha x_n + \beta y_n$	$\alpha X(e^{j\omega}) + \beta Y(e^{j\omega})$
Shift in time	x_{n-n_0}	$e^{-j\omega n_0}\, X(e^{j\omega})$
Shift in frequency	$e^{j\omega_0 n}\, x_n$	$X(e^{j(\omega-\omega_0)})$
Scaling in time		
Downsampling	x_{Nn}	$\frac{1}{N}\sum_{k=0}^{N-1} X(e^{j(\omega-2\pi k)/N})$
Upsampling	$x_{n/N}$, $n = \ell N$; 0, otherwise	$X(e^{jN\omega})$
Time reversal	x_{-n}	$X(e^{-j\omega})$
Differentiation in freq.	$(-jn)^k\, x_n$	$\frac{\partial^k X(e^{j\omega})}{\partial \omega^k}$
Moments	$m_k = \sum_{n\in\mathbb{Z}} n^k\, x_n = (-j)^k \left.\frac{\partial X(e^{j\omega})}{\partial\omega}\right\vert_{\omega=0}$	
Convolution in time	$(h * x)_n$	$H(e^{j\omega})\, X(e^{j\omega})$
Circular convolution in frequency	$h_n\, x_n$	$\frac{1}{2\pi}\,(H \circledast X)(e^{j\omega})$
Deterministic autocorrelation	$a_n = \sum_{k\in\mathbb{Z}} x_k\, x^*_{k-n}$	$A(e^{j\omega}) = \vert X(e^{j\omega})\vert^2$
Deterministic crosscorrelation	$c_n = \sum_{k\in\mathbb{Z}} x_k\, y^*_{k-n}$	$C(e^{j\omega}) = X(e^{j\omega})\, Y^*(e^{j\omega})$
Parseval's equality	$\Vert x\Vert^2 = \sum_{n\in\mathbb{Z}} \vert x_n\vert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \vert X(e^{j\omega})\vert^2\, d\omega = \frac{1}{2\pi}\,\Vert X\Vert^2$	
Related sequences		
Conjugate	x^*_n	$X^*(e^{-j\omega})$
Conjugate, time reversed	x^*_{-n}	$X^*(e^{j\omega})$
Real part	$\Re(x_n)$	$(X(e^{j\omega}) + X^*(e^{-j\omega}))/2$
Imaginary part	$\Im(x_n)$	$(X(e^{j\omega}) - X^*(e^{-j\omega}))/(2j)$
Conjugate-symmetric part	$(x_n + x^*_{-n})/2$	$\Re(X(e^{j\omega}))$
Conjugate-antisymmetric part	$(x_n - x^*_{-n})/(2j)$	$\Im(X(e^{j\omega}))$
Symmetries for real x		
X conjugate symmetric		$X(e^{j\omega}) = X^*(e^{-j\omega})$
Real part of X even		$\Re(X(e^{j\omega})) = \Re(X(e^{-j\omega}))$
Imaginary part of X odd		$\Im(X(e^{j\omega})) = -\Im(X(e^{-j\omega}))$
Magnitude of X even		$\vert X(e^{j\omega})\vert = \vert X(e^{-j\omega})\vert$
Phase of X odd		$\arg X(e^{j\omega}) = -\arg X(e^{-j\omega})$
Common transform pairs		
Kronecker delta sequence	δ_n	1
Shifted Kronecker delta sequence	δ_{n-n_0}	$e^{-j\omega n_0}$
Constant sequence	1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Geometric sequence	$\alpha^n u_n$	$1/(1-\alpha e^{-j\omega})$, $\vert\alpha\vert < 1$
Arithmetic-geometric sequence	$n\,\alpha^n u_n$	$\alpha e^{-j\omega}/(1-\alpha e^{-j\omega})^2$, $\vert\alpha\vert < 1$
Sinc sequence (ideal lowpass filter)	$\sqrt{\frac{\omega_0}{2\pi}}\,\mathrm{sinc}(\omega_0 n/2)$	$\begin{cases} \sqrt{2\pi/\omega_0}, & \vert\omega\vert \le \omega_0/2; \\ 0, & \text{otherwise.} \end{cases}$
Box sequence	$\begin{cases} 1/\sqrt{n_0}, & \vert n\vert \le (n_0-1)/2; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{n_0}\,\frac{\mathrm{sinc}\,(\omega n_0/2)}{\mathrm{sinc}\,(\omega/2)}$

Fourier transform properties

FT properties	Time domain	FT domain
Basic properties		
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\omega) + \beta Y(\omega)$
Shift in time	$x(t - t_0)$	$e^{-j\omega t_0}\, X(\omega)$
Shift in frequency	$e^{j\omega_0 t}\, x(t)$	$X(\omega - \omega_0)$
Scaling in time and frequency	$x(\alpha t)$	$(1/\alpha) X(\omega/\alpha)$
Time reversal	$x(-t)$	$X(-\omega)$
Differentiation in time	$d^n x(t)/dt^n$	$(j\omega)^n X(\omega)$
Differentiation in frequency	$(-jt)^n x(t)$	$d^n X(\omega)/d\omega^n$
Integration in time	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$X(\omega)/j\omega,\ X(0) = 0$
Moments	$m_k = \int_{-\infty}^{\infty} t^k x(t)\, dt = (j)^k \left.\frac{d^k X(\omega)}{d\omega^k}\right\rvert_{\omega=0}$	
Convolution in time	$(h * x)(t)$	$H(\omega)\, X(\omega)$
Convolution in frequency	$h(t)\, x(t)$	$\frac{1}{2\pi}(H * X)(\omega)$
Deterministic autocorrelation	$a(t) = \int_{-\infty}^{\infty} x(\tau)\, x^*(\tau - t)\, d\tau$	$A(\omega) = \lvert X(\omega)\rvert^2$
Deterministic crosscorrelation	$c(t) = \int_{-\infty}^{\infty} x(\tau)\, y^*(\tau - t)\, d\tau$	$C(\omega) = X(\omega) Y^*(\omega)$
Parseval's equality	$\lVert x\rVert^2 = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\, d\omega = \frac{1}{2\pi}\lVert X\rVert^2$	
Related functions		
Conjugate	$x^*(t)$	$X^*(-\omega)$
Conjugate, time reversed	$x^*(-t)$	$X^*(\omega)$
Real part	$\Re(x(t))$	$(X(\omega) + X^*(-\omega))/2$
Imaginary part	$\Im(x(t))$	$(X(\omega) - X^*(-\omega))/(2j)$
Conjugate-symmetric part	$(x(t) + x^*(-t))/2$	$\Re(X(\omega))$
Conjugate-antisymmetric part	$(x(t) - x^*(-t))/(2j)$	$\Im(X(\omega))$
Symmetries for real x		
X conjugate symmetric		$X(\omega) = X^*(-\omega)$
Real part of X even		$\Re(X(\omega)) = \Re(X(-\omega))$
Imaginary part of X odd		$\Im(X(\omega)) = -\Im(X(-\omega))$
Magnitude of X even		$\lvert X(\omega)\rvert = \lvert X(-\omega)\rvert$
Phase of X odd		$\arg X(\omega) = -\arg X(-\omega)$
Common transform pairs		
Dirac delta function	$\delta(t)$	1
Shifted Dirac delta function	$\delta(t - t_0)$	$e^{-j\omega_0 t}$
Dirac comb	$\sum_{n\in\mathbb{Z}} \delta(t - nT)$	$(2\pi/T)\sum_{k\in\mathbb{Z}} \delta(\omega - (2\pi/T)k)$
Constant function	1	$2\pi\, \delta(\omega)$
Exponential function	$e^{-\alpha\lvert t\rvert}$	$(2\alpha)/(\omega^2 + \alpha^2)$
Gaussian function	$e^{-\alpha t^2}$	$\sqrt{\pi/\alpha}\, e^{-\omega^2/\alpha}$
Sinc function (ideal lowpass filter)	$\sqrt{\frac{\omega_0}{2\pi}}\ \mathrm{sinc}(\omega_0 t/2)$	$\begin{cases} \sqrt{2\pi/\omega_0}, & \lvert\omega\rvert \le \omega_0/2; \\ 0, & \text{otherwise.} \end{cases}$
Box function	$\begin{cases} 1/\sqrt{t_0}, & \lvert t\rvert \le t_0/2; \\ 0, & \text{otherwise.} \end{cases}$	$\sqrt{t_0}\ \mathrm{sinc}\,(\omega t_0/2)$
Triangle function	$\begin{cases} 1 - \lvert t\rvert, & \lvert t\rvert < 1; \\ 0, & \text{otherwise.} \end{cases}$	$(\mathrm{sinc}\,(\omega/2))^2$